Duality, quantum skyrmions, and the stability of a SO(3) two-dimensional quantum spin glass

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Quantum topological excitations (skyrmions) are analyzed from the point of view of their duality to spin excitations in the different phases of a disordered two-dimensional, short-range interacting, SO(3) quantum magnetic system of Heisenberg type. The phase diagram displays all the phases, which are allowed by the duality relation. We study the large-distance behavior of the two-point correlation function of quantum skyrmions in each of these phases and, out of this, extract information about the energy spectrum and nontriviality of these excitations. The skyrmion correlators present a power-law decay in the spin-glass (SG) phase, indicating that these quantum topological excitations are gapless but nontrivial in this phase. The SG phase is dual to the AF phase, in the sense that topological and spin excitations are, respectively, gapless in each of them. The Berezinskii-Kosterlitz-Thouless mechanism guarantees the survival of the SG phase at $T\neq 0$, whereas the AF phase is washed out to $T=0$ by the quantum fluctuations. Our results suggest a more symmetric way of characterizing a SG phase: one for which both the order and disorder parameters vanish, namely, $\langle\sigma\rangle=0$ and $\langle\mu\rangle=0$, where σ is the spin and μ is the topological excitation operators.

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I. INTRODUCTION

Given a physical system, the spectrum of its possible excitations can, in principle, be roughly divided into two groups: Hamiltonian and topological excitations. The first ones correspond to states, which are created out of the ground state by the action of operators that appear explicitly in the Hamiltonian. These excitations carry in general quantum numbers corresponding to physical quantities, such as charge for instance, which are conserved as a consequence of some continuous symmetry of the Hamiltonian. Topological excitations, conversely bear quantum numbers that correspond to quantities whose conservation derives from a nontrivial topology of the space of classical configurations and whose continuity equation is satisfied as an identity rather then being the result of some continuous symmetry in the system.

In this work, we investigate the correlation functions of quantum skyrmions, which are the topological excitations that may occur in two-dimensional (2D) magnetic systems described by a Hamiltonian of the Heisenberg type, with SO(3) symmetry and nearest-neighbor interactions on a square lattice. We consider the quenched disordered case, with a Gaussian random distribution of exchange couplings centered at an antiferromagnetic (AF) coupling $\bar{J}>0$ with variance ΔJ.

This disordered system has been studied by means of a mapping onto a generalized nonlinear sigma model (NLSM), in which the original (staggered) spin is mapped onto the NLSM field $\mathbf{n}^{\alpha}$ (Refs. 1 and 2) and the index $\alpha=1,\ldots,n$ corresponds to the different replicas which are required for a quenched average. Eventually we must take the limit $n\rightarrow 0$.[3–5]

A remarkable feature of this system is that, whereas the quantum fluctuations completely "wash out," the ordered Néel phase at $T\neq 0$, in agreement with the Mermin-Wagner theorem,[6] the spin-glass phase persists even at a finite temperature. Even though this does not violate the theorem, because there is no spatial long-range order in the spin-glass (SG) phase, it is somewhat intriguing to have a SG phase at a finite temperature in a quantum 2D system, specially if we consider a lot of evidence against the occurrence of a SG phase at a finite temperature in 2D Ising systems.[7] We will see below, however, as a consequence of our study of quantum topological correlation functions, that this fact can be understood as a manifestation of the Berezinskii-Kosterlitz-Thouless (BKT) mechanism.[8,9] Indeed, it is the BKT mechanism that allows the existence of a SG phase at a finite temperature in this 2D system. Moreover, the action of this mechanism is only possible in the case of SO(3) systems, where a vortex picture for the skyrmions does exist. This explains why the corresponding SG phase is not found in 2D Ising systems, where such a picture is absent.

II. DUALITY AND QUANTUM TOPOLOGICAL EXCITATIONS

The nontriviality of the topology of classical configurations space is ultimately responsible for the stability of classical topological excitations. At the quantum level, this nontrivial topology manifests as a degeneracy of the ground state. The existence of ground-state degeneracy, therefore, is the indication that, at a quantum-mechanical level, the system presents quantum topological excitations in its spectrum. These excitations, differently from the former, cannot be created by acting on the ground state with Hamiltonian operators. A familiar example of topological excitations is magnetic vortices in 2D, which occur in type II superconductors. The corresponding topological "charge" would be the magnetic flux piercing the plane.

Topological excitations, surprisingly, are related to the concept of order-disorder duality, which plays an important role in many areas of physics. This can be seen in the following way. Consider a system characterized by a dynamical variable σ. It could be, for instance, the magnetization on a ferromagnetic system or the staggered magnetization in an antiferromagnetic one. Suppose the Hamiltonian $H[\sigma]$ possesses a symmetry, which at the quantum level is implemented by an unitary operator U such that, for g being the element of the symmetry group, $U\sigma U^\dagger = g\sigma$, with $[H,U]=0$. Since

$$\langle\sigma\rangle = \langle 0|\sigma|0\rangle = g\langle 0|U^\dagger \sigma U|0\rangle, \tag{2.1}$$

it follows that if $\langle\sigma\rangle \neq 0$, then necessarily $U|0\rangle = |0'\rangle \neq |0\rangle$, that is, the ground state will be degenerate. This, however, as remarked above, is a sign of nontriviality of the topological excitations. Now, assume the one-particle quantum topological excitation state is given by $|\mathrm{Top}\rangle = \mu|0\rangle$, where μ is the operator that creates these excitations out of the ground state. Then, if these states are nontrivial, we must have $\langle 0|\mu|0\rangle = 0$ because this means $|\mathrm{Top}\rangle$ is orthogonal to the ground state, as a genuine excited state must be. We conclude, therefore, that $\langle\sigma\rangle \neq 0$ implies $\langle\mu\rangle = 0$.

Conversely if $\langle 0|\mu|0\rangle \neq 0$, this would mean that $|\mathrm{Top}\rangle$ is not orthogonal to the ground state and hence is actually not a genuine excitation. This would imply the ground state should be unique because otherwise topological excitations should exist as nontrivial states. Now, if the ground state is unique, we have $U|0\rangle = |0\rangle$ and Eq. (2.1) would imply $\langle\sigma\rangle = 0$. We therefore conclude that $\langle\mu\rangle \neq 0$ implies $\langle\sigma\rangle = 0$.

The previous analysis shows that, if $\langle\sigma\rangle$ measures the amount of order (an order parameter) then $\langle\mu\rangle$ measures the amount of disorder, being naturally a disorder parameter. We see that a duality relation exists between the topological excitations creation operator μ and the Hamiltonian operator σ. The physical and mathematical properties of this duality relation are captured by the so-called dual algebra satisfied by σ and μ,[10] namely,

$$\mu(\mathbf{x},t)\sigma(\mathbf{y},t) = g(\mathbf{y}-\mathbf{x})\sigma(\mathbf{y},t)\mu(\mathbf{x},t), \tag{2.2}$$

where for each fixed $\mathbf{x}$ and $\mathbf{y}$, $g(\mathbf{x}-\mathbf{y})$ is an element of the symmetry group. This relation is the basis for constructing the topological excitations creation operator μ. It implies, for instance,

$$\langle\sigma\rangle\langle\mu\rangle = 0, \tag{2.3}$$

hence, we cannot have both $\langle\sigma\rangle$ and $\langle\mu\rangle$ nonvanishing. It also implies the spectrum is gapless whenever both $\langle\sigma\rangle = 0$ and $\langle\mu\rangle = 0$.[11] According to Eq. (2.3), for these kind of systems (with just one scalar order parameter) we can basically have only three phases. One with $\langle\sigma\rangle \neq 0$ and $\langle\mu\rangle = 0$, another with $\langle\sigma\rangle = 0$ and $\langle\mu\rangle \neq 0$, and finally a phase with both $\langle\sigma\rangle = 0$ and $\langle\mu\rangle = 0$. From the large-distance behavior of the quantum topological correlators, we show that the three phases allowed by the dual algebra are realized in the disordered system considered below.

III. THE CP^1 FORMULATION OF THE SHORT-RANGE AF HEISENBERG SPIN GLASS

A. Nonlinear sigma model formulation

We review in this and in the next subsection the field-theoretical description of the disordered SO(3) quantum Heisenberg-type system, which was studied with great detail in Refs. 1 and 2. It is described by the Hamiltonian operator,

$$\hat{\mathcal{H}} = \sum_{\langle ij\rangle} J_{ij} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j, \tag{3.1}$$

with nearest-neighbor interactions on a 2D square lattice of spacing a, having the random couplings J_{ij} associated with a Gaussian probability distribution with variance ΔJ and centered in $\bar{J} > 0$, namely,

$$P[J_{ij}] = \frac{1}{\sqrt{2\pi(\Delta J)^2}} \exp\left[-\frac{(J_{ij}-\bar{J})^2}{2(\Delta J)^2}\right]. \tag{3.2}$$

We consider the case of quenched disorder, which is conveniently dealt with the replica method.[3,4] In this case the average free energy is given by

$$\bar{F} = -k_B T \lim_{n\to 0} \frac{1}{n}([Z^n]_{av} - 1), \tag{3.3}$$

where $[Z^n]_{av}$ is the disorder-averaged replicated partition function.

With the help of spin coherent states $|\mathbf{\Omega}_i^\alpha(\tau)\rangle$ such that

$$\langle \mathbf{\Omega}_i^\alpha(\tau)|\hat{\mathbf{S}}_i^\alpha|\mathbf{\Omega}_i^\alpha(\tau)\rangle = S\mathbf{\Omega}_i^\alpha(\tau), \tag{3.4}$$

where S is the spin quantum number, it was shown[1] that in the continuum limit, the average replicated partition function corresponding to Eq. (3.1) could be written as

$$[Z^n]_{av} = \int \mathcal{D}\mathbf{n}\mathcal{D}Q\mathcal{D}\lambda \, \exp\left\{-\int d\tau \mathcal{L}_{\bar{J},\Delta}[\mathbf{n}^\alpha, Q^{\alpha\beta}, \lambda^\alpha]\right\}, \tag{3.5}$$

where the corresponding Lagrangian density is a generalized relativistic NLSM

$$\begin{aligned}\mathcal{L}_{\bar{J},\Delta} = {}& \frac{1}{2}|\nabla \mathbf{n}^\alpha|^2 + \frac{1}{2c^2}|\partial_\tau \mathbf{n}^\alpha|^2 + i\lambda_\alpha(|\mathbf{n}^\alpha|^2 - \rho_s) \\ & + \frac{D}{2}\int d\tau' \left[Q_{ab}^{\alpha\beta}(\tau,\tau') Q_{ab}^{\alpha\beta}(\tau,\tau') \right. \\ & \left. - \frac{2}{\rho_s} n_a^\alpha(\tau) Q_{ab}^{\alpha\beta}(\tau,\tau') n_b^\beta(\tau') \right].\end{aligned} \tag{3.6}$$

where $D = S^4(\Delta J)^2/a^2$ (a is the lattice parameter) and $\rho_s = S^2\bar{J}$. In the above expression, summation on the replica indices $\alpha, \beta = 1, \ldots, n$ is understood.

The field $\mathbf{n}^\alpha = (\sigma^\alpha, \vec{\pi}^\alpha)$ is the continuum limit of the (staggered) spin $\mathbf{\Omega}^\alpha$ and satisfies the constraint $\mathbf{n}^\alpha \cdot \mathbf{n}^\alpha = \rho_s$, which is implemented by integration on λ^α.

Decomposing $Q^{\alpha\beta}$ into replica diagonal and off-diagonal parts,

$$Q_{ab}^{\alpha\beta}(\vec{r};\tau,\tau') \equiv \delta_{ab}[\delta^{\alpha\beta}\chi(\vec{r};\tau,\tau') + q^{\alpha\beta}(\vec{r};\tau,\tau')], \tag{3.7}$$

where $q^{\alpha\beta} = 0$ for $\alpha = \beta$, we get

$$\mathcal{L}_{\bar{J},\Delta} = \frac{1}{2}|\nabla \mathbf{n}^\alpha|^2 + \frac{1}{2c^2}|\partial_\tau \mathbf{n}^\alpha|^2 + i\lambda_\alpha(|\mathbf{n}^\alpha|^2 - \rho_s) + \frac{3D}{2}\int d\tau' \bigg[n\chi^2(\tau,\tau') + q^{\alpha\beta}(\tau,\tau')q^{\alpha\beta}(\tau,\tau') - \frac{D}{\rho_s}\mathbf{n}^\alpha(\tau)\chi(\tau,\tau')\mathbf{n}^\alpha(\tau') - \frac{D}{\rho_s}\mathbf{n}^\alpha(\tau)q^{\alpha\beta}(\tau,\tau')\mathbf{n}^\beta(\tau')\bigg]. \tag{3.8}$$

This was the starting point for the CP^1 formulation, which was derived in Ref. 2. In a previous work,[1] we took a different path: from Eq. (3.8), we integrated over the $\vec{\pi}$ field and thereby obtained an effective action for the remaining fields. This allowed the determination of the average free energy and, out of this, the phase diagram of the system. Identical results were found within the CP^1 framework.[2]

B. CP^1 formulation

Introducing the CP^1 field as usual, by the relation

$$\mathbf{n}^\alpha(\tau) = \frac{1}{\sqrt{\rho_s}}[z_i^{*\alpha}(\tau)\sigma_{ij}z_j^\alpha(\tau)], \tag{3.9}$$

where the z_i^α field satisfies the constraint

$$|z_1^\alpha|^2 + |z_2^\alpha|^2 = \rho_s \tag{3.10}$$

and using the correspondence

$$\frac{1}{2}|\nabla \mathbf{n}^\alpha|^2 + \frac{1}{2c^2}|\partial_\tau \mathbf{n}^\alpha|^2 \Leftrightarrow 2\sum_{i=1}^{2}|D_\mu z_i^\alpha|^2, \tag{3.11}$$

($D_\mu=\partial_\mu+iA_\mu$), which comprises a functional integration over the auxiliary vector field A_μ, we obtain[2]

$$[Z^n]_{av} = \int \mathcal{D}z\mathcal{D}z^*\mathcal{D}A_\mu\mathcal{D}\chi\mathcal{D}q\mathcal{D}\lambda e^{-S}, \tag{3.12}$$

where $S[z_i^\alpha, z_i^{\alpha*}, A_\mu, \lambda, \chi(\tau,\tau'), q^{\alpha\beta}(\tau,\tau')]$ is the action corresponding to the Lagrangian density

$$\mathcal{L}_{\bar{J},\Delta,\mathbf{CP}^1} = 2|D_\mu z_i^\alpha|^2 + i\lambda_\alpha(|z_i^\alpha|^2 - \rho_s) + \frac{3D}{2}\int d\tau'[n\chi^2(\tau,\tau') + q^{\alpha\beta}(\tau,\tau')q^{\alpha\beta}(\tau,\tau')] + \frac{2D}{\rho_s^2}\int d\tau'\{[\chi(\tau,\tau')] \times [|z_i^{*\alpha}(\tau)|^2|z_j^\alpha(\tau')|^2] - [z_i^{*\alpha}z_j^\alpha(\tau)][\chi(\tau,\tau')\delta^{\alpha\beta} + q^{\alpha\beta}(\tau,\tau')][z_i^\beta z_j^{*\beta}(\tau')]\}, \tag{3.13}$$

where summation in i,j,α,β is understood.

In Ref. 2, we have determined the average free energy, by expanding the fields around their stationary point in Eq. (3.12) and integrating the quadratic quantum fluctuations of the z_i^α fields, namely,

$$S[z_i^\alpha, z_i^{\alpha*}, A_\mu, \lambda_\alpha, \chi, q^{\alpha\beta}] = S[z_{i,s}^\alpha, z_{i,s}^{\alpha*}, A_\mu^s, m^2, \chi_s, q_s^{\alpha\beta}] + \frac{1}{2}\int d\tau d\tau' \, \eta_i^{\alpha*}(\tau)\mathbb{M}_{ij}^{\alpha\beta}(\tau,\tau')\eta_j^\beta(\tau'), \tag{3.14}$$

where $\eta_i^\alpha = z_i^\alpha - z_{i,s}^\alpha$ and $\mathbb{M}$ is the matrix

$$\mathbb{M} = \begin{pmatrix} \dfrac{\delta^2 S}{\delta z_i^\alpha(\tau)\delta z_j^{*\beta}(\tau')} & \dfrac{\delta^2 S}{\delta z_i^\alpha(\tau)\delta z_j^\beta(\tau')} \\ \dfrac{\delta^2 S}{\delta z_i^{*\alpha}(\tau)\delta z_j^{*\beta}(\tau')} & \dfrac{\delta^2 S}{\delta z_i^{*\alpha}(\tau)\delta z_j^\beta(\tau')} \end{pmatrix}, \tag{3.15}$$

with elements taken at the stationary fields.

Inserting Eq. (3.14) in Eq. (3.12) we obtain, after integrating over the z fields,

$$[Z^n]_{av} = e^{-nS_{\text{eff}}[\sigma_s^\alpha, m^2, A_\mu^s, q_s^{\alpha\beta}(\tau-\tau'), \chi_s(\tau-\tau')]}, \tag{3.16}$$

where

$$S_{\text{eff}} = S[\sigma_s^\alpha, m^2, A_\mu^s, q_s^{\alpha\beta}, \chi_s] - \frac{1}{n}\ln \text{Det } \mathbb{M}, \tag{3.17}$$

We conclude, because of Eq. (3.3), that

$$\bar{F} = \frac{1}{\beta}S_{\text{eff}}[\sigma_s^\alpha, m^2, A_\mu^s, q_s^{\alpha\beta}(\tau-\tau'), \chi_s(\tau-\tau')]. \tag{3.18}$$

In the previous expressions, the subscript s means that the fields are taken at their stationary values: λ_s ($m^2=2i\lambda_s$ is the spin gap), $A_{s,\mu}=0$, $\chi_s(\tau-\tau')$ and $q_s^{\alpha\beta}(\tau-\tau')$.

The staggered magnetization σ_s^α is given in terms of the CP^1 fields as

$$\sigma_s^2 = \frac{1}{n}\sum_{\alpha=1}^{n}[|z_{1,s}^\alpha|^2 + |z_{2,s}^\alpha|^2] \equiv \frac{1}{n}\sum_{\alpha=1}^{n}\sigma_\alpha^2. \tag{3.19}$$

From the average free energy one can derive the phase diagram of the system, which is shown in Fig. 1.[1,2] This presents a critical line separating the SG and paramagnetic (PM) phases, which starts, at $T=0$, in the quantum critical point

$$\rho_0 = \frac{\Lambda}{2\pi}\left[1 + \frac{1}{\gamma}\left[1 + \frac{1}{2}\ln(1+\gamma)\right]\right], \tag{3.20}$$

where

$$\gamma = 3\pi\left(\frac{\bar{J}}{\Delta J}\right)^2 = \frac{3\pi\rho_s^2\Lambda^2}{D} \tag{3.21}$$

and $\Lambda=1/a$ is the high-momentum cutoff. For $\rho>\rho_0$, there is an ordered (AF) Néel phase at $T=0$.

The Edwards-Anderson (EA) order parameter, which is used for detecting the occurrence of a SG phase,[3,4] is given by

$$q_{\text{EA}} = T\bar{q}_0, \tag{3.22}$$

where

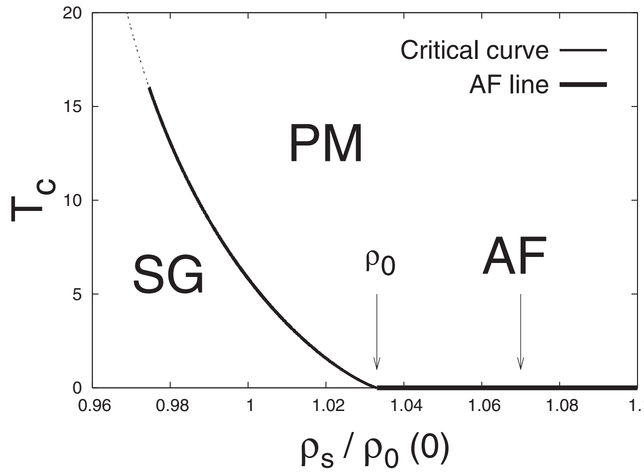

FIG. 1. Phase diagram for $\gamma = 10^2$ and $\Lambda = 10^3$. The critical curve valid near the QCP ρ_0 (solid curve). $\rho_0(0) = \Lambda / 2\pi$ is the quantum critical point of the pure AF system. The value ascribed to Λ is a realistic one in K ($\Lambda \rightarrow \frac{\hbar v_s}{k_B}\Lambda$; v_s: spin-wave velocity).

$$\bar{q}_0 = \frac{1}{n(n-1)} \sum_{\alpha,\beta} q^{\alpha\beta}(\omega_n = 0).$$

(ω_n are the Matsubara frequencies).

A detailed study of the phase diagram of the system, explicitly showing the $\bar{J}$ and ΔJ dependence can be found in Ref. 2. We would like to emphasize that in obtaining the above phase diagram, both in the CP^1 and NLSM versions, quantum fluctuations have been included in the derivation of the average free energy, hence it goes beyond the mean-field approximation. The fact that the ordered AF phase is washed out from any $T \neq 0$ is a clear evidence for this.

Our extended NLSM depends on the coupling $\bar{J}$ precisely in the same way as the usual NLSM. Thus, the analysis of the validity of the approximations made, as a function of the value of this coupling follows closely the corresponding analysis well known in the simple NLSM.[12]

IV. QUANTUM SKYRMION CORRELATION FUNCTIONS

A. General method

Given a theory containing an abelian gauge field in two dimensions, the topological excitations are magnetic vortices.[13,14] Skyrmions, which are topological excitations of the NLSM, accordingly, appear as magnetic (in the A_μ field) vortices in the CP^1 formulation. The correlation functions of the corresponding vortex quantum creation operator $\mu(\vec{x},\tau)$ are obtained by treating this operator as a disorder variable, dual to the order parameter of the system, as we saw in Sec. II. Then, a method of quantization has been developed,[13,14] where all correlation functions of the topological excitation creation operator, can be obtained by modifying the integrand of the partition function by adding to the corresponding field intensity tensor $F_{\mu\nu}$, an external particular field configuration $\tilde{B}_{\mu\nu}(z;x,y)$. We implement this method below for the present case.

We will take the CP^1 field z_i as the "order" field. From Eq. (3.19), it is clear that whenever $\langle z_i^\alpha \rangle \neq 0$, the staggered magnetization will not vanish as well. We can then introduce the dual algebra relating z_i^α with the topological excitation creation operator, in the form of Eq. (2.2). Using the fact that the symmetry group of the model is U(1), we have[13]

$$\mu(\mathbf{x},t) z_i^\alpha(\mathbf{y},t) = \exp\{i \arg(\mathbf{y}-\mathbf{x})\} z_i^\alpha(\mathbf{y},t) \mu(\mathbf{x},t). \quad (4.1)$$

The vortex creation operator μ satisfying Eq. (4.1) is given by[13]

$$\mu(\mathbf{x},t) = \exp\left\{ i2\pi \int d^2\mathbf{r} \arg(\mathbf{r}-\mathbf{x}) \sum_{\alpha=1}^{n} [z_i^{\alpha *}\pi_i^{\alpha *} - z_i^\alpha \pi_i^\alpha] \times (\mathbf{r},t) \right\}, \quad (4.2)$$

where π_i^α is the momentum canonically conjugate to z_i^α. Whenever the A_μ-field kinematics is described by a Maxwell term, this can also be written as

$$\mu(\mathbf{x},t) = \exp\left\{ i2\pi \int d^2\mathbf{r} \arg(\mathbf{r}-\mathbf{x}) \partial_i F^{0i}(\mathbf{r},t) \right\},$$

by just considering the field equation. Then, using the Cauchy-Riemann equation for $\arg(\mathbf{r}-\mathbf{x})$ and the analytical properties of this function, we obtain, equivalently

$$\mu(\mathbf{x},t) = \exp\left\{ i2\pi \int_{(\mathbf{x},t)}^{\infty} d\xi_i \epsilon^{ij} F^{0j}(\mathbf{r},t) \right\}, \quad (4.3)$$

where $F^{\mu\nu}$ is the field intensity tensor corresponding to A_μ.

This last form of the operator is the most useful for obtaining the correlation functions. It is not difficult to infer from Eq. (4.3) that, for a Maxwell-type action, for which S_g is quadratic in $F^{\mu\nu}$, the μ-field two-point correlation function will be given by[13]

$$\langle \mu(\mathbf{x},\tau)\mu^\dagger(\mathbf{y},\tau') \rangle = \int \mathcal{D}A_\mu \exp\{-S_g[F_{\mu\nu} + \tilde{B}_{\mu\nu}(z;x,y)]\}, \quad (4.4)$$

where

$$\tilde{B}_{\mu\nu}(z;x,y) = 2\pi \int_{x=(\mathbf{x},\tau)}^{y=(\mathbf{y},\tau')} d\xi_\lambda \epsilon^{\lambda\mu\nu} \delta(z-\xi). \quad (4.5)$$

This turns out to be a general expression for the topological excitation correlators, namely, it holds for any action $S_g[F_{\mu\nu}]$ whatsoever, irrespective of its form.[14,13]

Now, an important adjustment must be made, in order to adapt this result to the present disordered system. In order to recover the physical thermodynamics, we must take the limit where the number of replicas vanishes, $n \rightarrow 0$, hence we must be careful when defining the physical correlator in this limit. From Eq. (4.2), we see that we actually have

$$\langle \mu\mu^{\dagger}\rangle = \prod_{\alpha=1}^{n} \langle \mu\mu^{\dagger}\rangle_{\alpha}, \tag{4.6}$$

because fields belonging to different replicas commute. It is natural, therefore, to define the physical correlation function as the geometrical average among the n replicas, before taking the limit $n \to 0$, namely,

$$\langle \mu\mu^{\dagger}\rangle_{\mathrm{phys}} = \lim_{n\to 0}\left[\prod_{\alpha=1}^{n} \langle \mu\mu^{\dagger}\rangle_{\alpha}\right]^{1/n} = \lim_{n\to 0}\langle \mu\mu^{\dagger}\rangle^{1/n}, \tag{4.7}$$

where the correlator on the right-hand side is given by Eq. (4.4).

B. Effective gauge field theory

We must now determine the form of the A_μ-field action $S_g[F_{\mu\nu}]$ in Eq. (4.4), in order to calculate the topological quantum correlation function. For this purpose, we start from the CP^1 description of the system,[2] expand the action in the z_i's and A_μ fields up to the second order around the stationary points ($z_i^{\alpha}=A_{\mu}^{s}=0$) and perform the quadratic z_i integrals. We must stress that, expanding around $z_i^{\alpha}=0$, we are only considering the case when $\langle\sigma\rangle=0$ and therefore this analysis does not apply to the ordered AF phase. This has been already considered elsewhere.[15]

$$Z_{A_\mu} = \int \mathcal{D}z\mathcal{D}z^{*}\mathcal{D}A_{\mu} e^{-S[z_i^{\alpha},z_i^{\alpha *},A_{\mu},m^2,\chi_s(\tau-\tau'),q_s^{\alpha\beta}(\tau-\tau')]}. \tag{4.8}$$

We took the fields λ, χ and $q^{\alpha\beta}$ at their stationary point leaving the integrals in z_i and A_μ. Then, integrating on the z_i fields and expanding the action in the A_μ field up to quadratic fluctuations around the stationary configuration $A_{\mu}^{s}=0$, we get

$$Z_{A_\mu} = \int \mathcal{D}A_{\mu} e^{-S_g[A_\mu]}, \tag{4.9}$$

where

$$S_g[A_\mu] = \frac{n}{2}\int_0^{\beta} d\tau \int d^2\mathrm{r} A_{\mu}\left[\frac{\delta^2 S_{\mathrm{eff}}(A_\mu)}{\delta A_\mu \delta A_\nu}\right]_{A_\mu=0} A_\nu. \tag{4.10}$$

In Appendix A, we show that the action for the A_μ field can be written as

$$S_g[A_\mu] = n\int_0^{\beta} d\tau \int d^2\mathrm{r}\left\{\frac{\kappa}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{4}\int_0^{\beta} d\tau' \int d^2\mathrm{r}' F_{\mu\nu}(\mathrm{r},\tau)\tilde{\Pi}(\mathrm{r},\mathrm{r}';\tau,\tau')F^{\mu\nu}(\mathrm{r}',\tau')\right\}, \tag{4.11}$$

where $\kappa=\frac{m}{4\pi}$, $\alpha=\bar{q}_0(\frac{D}{2\pi\rho_s})$ and

$$\tilde{\Pi}(\vec{k},\omega_n) = \frac{T}{[|\vec{k}|^2+\omega_n^2]}\int_0^1 \frac{dx}{|\vec{k}|^2 x(1-x)+x\omega_n^2+m^2} \tag{4.12}$$

Notice that the α term in the effective action, Eq. (4.11), is different from zero only in the SG phase, where $\bar{q}_0 \neq 0$.

C. Skyrmion correlation functions

We may write Eq. (4.4) as

$$\langle \mu(\vec{x},\tau)\mu^{\dagger}(\vec{y},\tau')\rangle = \int \mathcal{D}A_{\mu} \exp\left\{-\int_0^{\beta}\int d^2\mathrm{r}\left[\frac{n}{2}A_{\mu}[-\Box\Pi]A_{\nu} + \sqrt{n}\partial_{\nu}\tilde{B}^{\nu\mu}[\Pi]A_{\mu} + \frac{1}{4}\tilde{B}_{\mu\nu}[\Pi]\tilde{B}_{\mu\nu}\right]\right\}, \tag{4.13}$$

where

$$\Pi = \kappa + \alpha\tilde{\Pi}. \tag{4.14}$$

Introducing a gauge fixing term, we can integrate Eq. (4.13) in A_μ, obtaining

$$\langle \mu(\vec{x},\tau)\mu^{\dagger}(\vec{y},\tau')\rangle_{\mathrm{phys}} = \exp\left\{2\pi^2\int_x^y d\xi_{\lambda}\int_x^y d\eta_{\rho}\epsilon^{\lambda\mu\alpha}\epsilon^{\rho\nu\alpha} \times \partial_{\mu}\partial'_{\nu}F(\vec{\xi}-\vec{\eta};\xi_0-\eta_0) - \frac{1}{4}\tilde{B}_{\mu\nu}[\Pi]\tilde{B}_{\mu\nu}\right\}, \tag{4.15}$$

where $x=(\mathbf{x},\tau), y=(\mathbf{y},\tau')$,

$$F(\vec{x};\tau) = T\sum_{\omega_n}\int \frac{d^2k}{(2\pi)^2}F(\vec{k},\omega_n)e^{i\vec{k}\cdot\vec{x}}e^{-i\omega_n\tau}, \tag{4.16}$$

and

$$F(\vec{k},\omega_n) = \frac{\Pi}{-\Box} = \frac{\kappa}{|\vec{k}|^2+\omega_n^2} + \frac{\alpha T}{[|\vec{k}|^2+\omega_n^2]^2}\int_0^1 \frac{dx}{|\vec{k}|^2 x(1-x)+x\omega_n^2+m^2}. \tag{4.17}$$

Using the identity

$$\epsilon^{\lambda\mu\alpha}\epsilon^{\rho\nu\alpha} = \delta^{\lambda\rho}\delta^{\mu\nu} - \delta^{\lambda\nu}\delta^{\mu\rho}$$

in Eq. (4.15) we can see that the first term cancels the last one in Eq. (4.13) [and in Eq. (4.15)] and the second one gives

$$\langle \mu(\vec{x},\tau)\mu^{\dagger}(\vec{y},\tau')\rangle_{\mathrm{phys}} = \exp\{4\pi^2[F(\vec{x}-\vec{y};\tau-\tau') - F(\vec{\epsilon};0)]\}, \tag{4.18}$$

where ϵ is a short-distance cutoff. Introducing the renormalized skyrmion field operator

$$\mu_R(\vec{x},\tau) = \exp\{2\pi^2 F(\vec{\epsilon};0)\}\mu_{(\vec{x},\tau)},$$

we get the renormalized and finite correlation function

$$\langle \mu_R(\vec{x},\tau)\mu_R^{\dagger}(\vec{y},\tau')\rangle_{\mathrm{phys}} = \exp\{4\pi^2 F(\vec{x}-\vec{y};\tau-\tau')\} \tag{4.19}$$

In Appendix B, we calculate the inverse Fourier transforms of the κ and α terms of the function F. These allow us to determine the large-distance behavior of the quantum skyrmion correlation functions, Eq. (4.18).

V. LARGE-DISTANCE BEHAVIOR OF SKYRMION CORRELATORS

A. Néel phase

The skyrmion two-point correlation function has been evaluated in the ordered AF phase, which occurs on the line $T=0;\rho_s>\rho_0$ in Ref. 15. It presents the following large-distance behavior

$$\langle \mu_R(\vec{x},\tau)\mu_R^\dagger(\vec{y},\tau)\rangle_{\rm phys}^{\rm AF} \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} \exp\{-2\pi\sigma^2|\vec{x}-\vec{y}|\}, \quad (5.1)$$

where σ is the staggered magnetization satisfying $\sigma^2=\frac{1}{8}[\rho_s-\rho_0]$.[1] The equation above implies

$$\langle \mu_R\rangle^{\rm AF}=0, \quad (5.2)$$

meaning that the quantum skyrmion states $|\mu_R\rangle^{\rm AF}$ are orthogonal to the ground state and are, consequently, nontrivial. The exponential decay of the skyrmion correlator, conversely, implies the corresponding quantum excitations (skyrmion) have a gap $E_g=2\pi\sigma^2$. Notice that this gap, which may be written as

$$E_g=\frac{\pi}{4}[\rho_s-\rho_0], \quad (5.3)$$

vanishes as we approach the quantum phase transition to the SG phase at the quantum critical point $(T=0,\rho_s=\rho_0)$.

B. Paramagnetic phase

In the PM phase, we have both $\sigma=0$ and $\bar{q}_0=0$ and, consequently, only the κ term of the function F in Eq. (4.17) contributes to the skyrmion correlation function, Eq. (4.18). We calculated this term, for equal times, in Appendix B. From Eq. (B.4) we have, at $T=0$,

$$\langle \mu_R(\vec{x},\tau)\mu_R^\dagger(\vec{y},\tau)\rangle_{\rm phys}^{\rm PM} \overset{T\to 0}{\longrightarrow} \exp\left\{\frac{\pi\kappa}{|\vec{x}-\vec{y}|}\right\}. \quad (5.4)$$

For an arbitrary temperature, T, we obtain the large-distance behavior [see Eq. (B.7)]

$$\langle \mu_R(\vec{x},\tau)\mu_R^\dagger(\vec{y},\tau)\rangle_{\rm phys}^{\rm PM} \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} \exp\left\{\frac{\pi\kappa}{|\vec{x}-\vec{y}|}\left[\frac{1}{2}+\frac{\pi T|\vec{x}-\vec{y}|}{\sqrt{2}}\coth(\sqrt{2}\pi T|\vec{x}-\vec{y}|)\right]\right\}. \quad (5.5)$$

From the above expression, we may infer that

$$\langle \mu_R\rangle^{\rm PM}=\exp\left\{\frac{\pi^2\kappa T}{2\sqrt{2}}\right\}\neq 0. \quad (5.6)$$

This nonzero result is the one to be expected in the PM phase where the topological excitations should not be genuine excitations, due to the absence of spontaneous symmetry breaking. The above result just confirms this fact, by stating that the topological excitation quantum state $|\mu_R\rangle^{\rm PM}$ is not orthogonal to the ground state, being therefore trivial.

C. Spin-glass phase

In the SG phase, we have $\sigma=0$ but now $\bar{q}_0\neq 0$. Then, the α term of the function F in Eq. (4.17) will contribute to the skyrmion correlation function, Eq. (4.18). We have calculated this term, for large distances and equal times, in Appendix B. This has a logarithmic behavior and therefore dominates the large-distance behavior of the function F. Indeed, according to Eq. (B.9), we have

$$F(\vec{x}-\vec{y};0) \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} -\frac{\bar{q}_0 D}{24\pi^2\rho_s m^2}\left[1-\frac{3T^2}{2m^2}\right]\ln C|\vec{x}-\vec{y}|, \quad (5.7)$$

where C is a constant.

Inserting this result in Eq. (4.19), we get

$$\langle \mu_R(\vec{x},\tau)\mu_R^\dagger(\vec{y},\tau)\rangle_{\rm phys}^{\rm SG} \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} \frac{1}{|\vec{x}-\vec{y}|^\nu}, \quad (5.8)$$

where

$$\nu=\frac{\bar{q}_0 D}{6\rho_s m^2}\left[1-\frac{3T^2}{2m^2}\right]. \quad (5.9)$$

In realistic systems, we always have $T^2\ll m^2$ (typically $T\simeq 10$ K and $m\simeq 100$ K, therefore ν is always positive in the SG phase. As a consequence of this

$$\langle \mu_R(\vec{x},\tau)\mu_R^\dagger(\vec{y},\tau)\rangle_{\rm phys}^{\rm SG} \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} 0 \quad (5.10)$$

and therefore

$$\langle \mu_R\rangle^{\rm SG}=0. \quad (5.11)$$

This result shows that the quantum skyrmion states are orthogonal to the ground state in the SG phase, being therefore nontrivial quantum states. The power-law behavior of their two-point correlator, however implies that they have a zero excitation gap. This is a quite interesting result. It reveals the existence of a duality relation between the AF and SG phases. The usual order-disorder duality occurs between the AF and PM phases, namely, the order and disorder parameters, $\langle\sigma\rangle$ and $\langle\mu\rangle$ are, respectively, nonzero in each of these two phases while the other vanishes.

The duality between the AF and SG phases, for both of which $q_{\rm EA}\neq 0$, however, is of a different nature. The spin excitations are gapless whereas the topological ones are gapped in the AF phase and conversely the spin excitations are gapped whereas the topological ones are gapless in the SG phase. Our finding of gapless topological excitations in the SG phase is a key step in establishing this duality relation.

D. Spin-glass phase and the BKT mechanism

There is also an important fact related to the existence of gapless topological excitations and their associate power-law correlators in the SG phase. When using the CP^1 language the skyrmions become vortices. The power-law behavior of their correlation functions is a clear indication that we have a Berezinskii-Kosterlitz-Thouless (BKT) two-dimensional system of vortices.[8] Indeed, quantum vortices do have a large-distance power-law decay in the low-temperature phase, which exists below the critical point in a BKT system.[9] This explains the existence of a SG phase at a finite temperature, as observed in Refs. 1 and 2: it is a BKT phase supporting gapless vortices. Conversely, the presence of gapless spin-wave excitations in the AF phase, washes this phase away at any finite temperature through the well-known Mermin-Wagner mechanism.[6] This is the explanation for the asymmetry found between the SG and AF phases of this system, the former persisting at a finite T, whereas the other only remains at $T=0$.

The operation of the BKT mechanism in this system only occurs because it is possible to use a CP^1 description, which presents gapless vortices in a SG phase. Thereby one can understand why we can have a stable SG phase in a quantum SO(3) disordered 2D Heisenberg system but not in the corresponding Ising system.[7] The latter does not allow a CP^1 formulation with the corresponding vortices and therefore cannot display the BKT mechanism.

E. Characterization of the spin-glass phase

The SG phase is the realization of one of the possible phases allowed by the dual algebra of spin and topological excitation operators, namely, the one where both $\langle\sigma\rangle=0$ and $\langle\mu\rangle=0$. This criterion, both order and disorder parameters vanishing, can be used as a more symmetrical alternative for the characterization of a SG phase then the usual one where $\langle\sigma\rangle=0$ and $q_{\mathrm{EA}}\neq 0$. We can state, equivalently that a SG phase is one where the quantum topological excitations are nontrivial gapless states of the Hilbert space.

One can speculate whether this is a general property or a peculiarity of the present model. From the point of view of energetics, it is clear that the creation of a skyrmion defect out of an ordered ground state, such as the one we have in an AF phase, costs a finite amount of energy since a number of spins must be flipped, in order to create the quantum-defect state. Conversely, creating such a defect on a disordered ground state, as the one we have in a paramagnetic phase, clearly does not change the state of the system and therefore costs no energy. The skyrmion operator actually does not create a truly new state. We can consider that the topological defects are condensed in the ground state, thereby producing the disordered PM state. Skyrmions here are not genuine excitations.

In a spin-glass phase the ground state is also a disordered state. Therefore, when creating a skyrmion defect on such a state, we may conclude that, as in the PM phase, there will be no cost in energy, because of the disordered character of the ground state. However, differently from the PM phase, the SG ground state is a "frozen" disordered state. Consequently, the new state generated by the inclusion of the skyrmion will be nontrivially different from the ground state, despite being also disordered and having zero energy cost for its creation. The skyrmion state, hence, must be orthogonal to the ground state, implying the correlation function must vanish at large distances. This way the large-distance behavior of the skyrmion correlation function detects the frozen nature of the ground state. The result is the occurrence of zero energy nontrivial topological states orthogonal to the ground state. The fact that they bear a nonzero topological charge clearly distinguishes them from the ground state.

These arguments may be generalized for Ising systems, for instance, by replacing skyrmions by Bloch walls and can be applied whenever topological defects may be introduced. It seems to allow a broad characterization of a SG phase, for a vast class of systems, as one for which both the order and disorder parameters vanish, namely, $\langle\sigma\rangle=0$ and $\langle\mu\rangle=0$, where σ is the spin and μ is the topological excitation operators.

VI. CONCLUSION

The disordered magnetic system considered in this work presents all the possible phases allowed by the duality relation, which exists between the staggered spin operator and the creation operator of quantum topological excitations, namely, quantum skyrmions. There is an ordered antiferromagnetic phase with $\langle\sigma\rangle\neq 0$ and $\langle\mu\rangle=0$, a paramagnetic phase with $\langle\sigma\rangle=0$ and $\langle\mu\rangle\neq 0$, and a spin-glass phase, with $\langle\sigma\rangle=0$ and $\langle\mu\rangle=0$.

The PM and AF phases are dual to each other, in the sense that the order and disorder parameters, respectively, $\langle\sigma\rangle$ and $\langle\mu\rangle$ show a complementary behavior, being zero or not, respectively, in each of the two phases. An interesting duality relation, however, also exists between the AF and SG phases, concerning the gap of topological and spin excitations. In the AF phase, we have gapless spin excitations, namely, $m=0$, whereas the quantum topological excitations have a finite gap proportional to σ^2 and given by Eq. (5.3). The SG phase, conversely, presents spin excitations with a nonvanishing gap, namely, $m\neq 0$,[1,2] whereas the topological excitations are gapless, according to Eq. (5.8). The AF and SG phases, therefore are dual with respect to the gap of the spin and topological excitations. The paramagnetic phase presents spin excitations with a gap $m\neq 0$, but there are no genuine topological excitations in the Hilbert space, since the creation operator of topological excitations acting on the ground state produces basically the same state.

The fact that in the SG phase the skyrmion state is orthogonal to the vacuum in spite of having zero energy is an indication that the ground state is frozen albeit disordered. We, therefore arrive at an alternative characterization of a spin-glass state, as one in which both the order and disorder parameters vanish and the quantum topological excitations in a magnetic systems are gapless but nontrivial.

Our results indicate that the stability of the SG phase, where both $\langle\sigma\rangle$ and $\langle\mu\rangle$ vanish, at a finite temperature, is a consequence of the BKT mechanism. This does not apply to Ising systems, where a vortex picture of topological excitations does not exist.

The average free energy of a spin glass is expected to have a large amount of local minima. The problem of determining the absolute minimum is a difficult one. One can find results in the literature, indicating that renormalization group flows may drive a replica symmetric solution toward a broken replica symmetry one. One possibility is that this would happen in a high-coupling $(\bar{J})$ limit. This would bring the system to a new local minimum where the replica symmetry would be broken. This is a very interesting subject for future investigation. It is, however beyond the scope of the present work.

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APPENDIX A: EFFECTIVE ACTION FOR A_μ

Let us derive here the expression for the action of the A_μ field. We will consider only the case $\sigma=0$, namely, the PM and SG phases. From Eq. (3.17) it follows, in this case that

$$\Pi^{\mu\nu} = \left[\frac{\delta^2 S_{\text{eff}}(A_\mu)}{\delta A_\mu \delta A_\nu}\right]_{A_\mu=0} = \frac{1}{n}\frac{\delta^2}{\delta A_\mu \delta A_\nu}[\ln \text{Det}\, \mathbb{M}(A_\mu)]_{A_\mu=0}. \tag{A.1}$$

By expanding in A_μ and taking the derivatives before making the limit $A_\mu \to 0$, we find in frequency-momentum space $k^\mu = (\omega_n, \vec{k})$,

$$\Pi^{\mu\nu}(\omega_n, \vec{k}) = \kappa(k^2\delta^{\mu\nu} - k^\mu k^\nu) + A\bar{q}_0\Gamma(\omega_n,\vec{k})\delta^{\mu\nu}, \tag{A.2}$$

where

$$\Gamma(\omega_n,\vec{k}) = T\int \frac{d^2q}{(2\pi)^2}\frac{1}{[|\vec{q}|^2+m^2][|\vec{k}-\vec{q}|^2+\omega_n^2+m^2]} = \frac{T}{4\pi}\int_0^1 \frac{dx}{|\vec{k}|^2 x(1-x) + x\omega_n^2 + m^2}. \tag{A.3}$$

Multiplying and dividing the last term in Eq. (A.2) by $k^2 = (|\vec{k}|^2 + \omega_n^2)$ and adding a pure gauge term, the result, Eq. (4.11), follows, with

$$\tilde{\Pi} = \frac{\Gamma}{k^2}.$$

APPENDIX B: FINITE-TEMPERATURE INVERSE FOURIER TRANSFORMS

1. Inverse transform of $\frac{1}{k^2+\omega_n^2}$

We have $\mathcal{F}^{-1}[\frac{1}{k^2+\omega_n^2}]$ given by

$$\mathcal{F}_1(\vec{x};\tau) = T\sum_{\omega_n}\int \frac{d^2k}{(2\pi)^2}\frac{e^{i(\vec{k}\cdot\vec{x}-i\omega_n\tau)}}{k^2+\omega_n^2}. \tag{B.1}$$

The $\vec{k}$ integral may be easily done,[16] yielding

$$\mathcal{F}_1(\vec{x};\tau) = \frac{T}{2\pi}\left\{-\lim_{\epsilon\to 0}\ln\frac{\epsilon}{2}|\vec{x}| + 2\sum_{n=1}^{\infty} K_0(|\omega_n||\vec{x}|)e^{-i\omega_n\tau}\right\}, \tag{B.2}$$

where K_0 is a modified Bessel function. For $\tau=0$ we can perform the sum.[16] The logarithmic term is canceled and we get, up to a constant [which will not contribute to the skyrmion correlation function, as we can infer from Eq. (4.18)]

$$\mathcal{F}_1(\vec{x};0) = \frac{1}{4\pi|\vec{x}-\vec{y}|}\left\{1 + T\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{T^2+\frac{n^2}{|\vec{x}-\vec{y}|^2}}} - \frac{1}{\sqrt{\frac{n^2}{|\vec{x}-\vec{y}|^2}}}\right]\right\}. \tag{B.3}$$

We immediately see that, for zero temperature we get

$$\mathcal{F}_1(\vec{x};0) \overset{T\to 0}{\rightarrow} \frac{1}{4\pi|\vec{x}-\vec{y}|}. \tag{B.4}$$

For finite temperatures, we obtain from Eq. (B.3), at large distances

$$\mathcal{F}_1(\vec{x};0) \overset{|\vec{x}-\vec{y}|\to\infty}{\rightarrow} \frac{1}{4\pi|\vec{x}-\vec{y}|}\left\{\frac{1}{2} + \frac{\pi T|\vec{x}-\vec{y}|}{\sqrt{2}}\coth[\sqrt{2}\pi T|\vec{x}-\vec{y}|]\right\}. \tag{B.5}$$

2. Inverse transform of $\frac{\tilde{\Pi}(\vec{k},\omega_n)}{[k^2+\omega_n^2]^2}$

Now we have $\mathcal{F}^{-1}[\frac{\tilde{\Pi}(\vec{k},\omega_n)}{[k^2+\omega_n^2]^2}]$ given by

$$\mathcal{F}_2(\vec{x};\tau) = T\sum_{\omega_n}\int \frac{d^2k}{(2\pi)^2}\frac{\tilde{\Pi}(|\vec{k}|,\omega_n)}{[k^2+\omega_n^2]^2}e^{i(\vec{k}\cdot\vec{x}-i\omega_n\tau)}. \tag{B.6}$$

Integrating on the angular $\vec{k}$ variable, we get

$$\mathcal{F}_2(\vec{x};\tau) = \frac{T}{4\pi}\sum_{\omega_n}\int_0^\infty dk\left[\frac{-1}{2}\frac{\partial}{\partial k}\left(\frac{1}{k^2+\omega_n^2}\right)\right]\tilde{\Pi}(|\vec{k}|,\omega_n) \times J_0(k|\vec{x}|)e^{-i\omega_n\tau}. \tag{B.7}$$

We are actually interested in the large-distance behavior of $\mathcal{F}_2(\vec{x};0)$. In this regime, only the $k\to 0$ will contribute to the k integral. Since $\tilde{\Pi}(|\vec{k}|,\omega_n)$ is always regular for $k\to 0$, we can replace it by $\tilde{\Pi}(0,\omega_n)$. Then the ω_n sum may be performed, for $\tau=0$, yielding to leading order in k

$$\mathcal{F}_2(\vec{x};0) \overset{|\vec{x}|\to\infty}{\rightarrow} \frac{1}{12\pi m^2}\left[1 - \frac{3T^2}{2m^2}\right]\lim_{\epsilon\to 0}\int_0^\infty dk k\frac{J_0(k|\vec{x}|)}{k^2+\epsilon^2}. \tag{B.8}$$

The last integral gives[16] $K_0(\epsilon|\vec{x}|) \overset{\epsilon\to 0}{\rightarrow} -\ln C|\vec{x}|$, hence

$$\mathcal{F}_2(\vec{x};0) \overset{|\vec{x}|\to\infty}{\rightarrow} -\frac{1}{12\pi m^2}\left[1 - \frac{3T^2}{2m^2}\right]\ln C|\vec{x}|. \tag{B.9}$$

This immediately leads to Eq. (5.7) and to the power-law behavior of the skyrmion correlation function in the SG phase, Eqs. (5.8) and (5.9). The constant C will not contribute to the skyrmion correlation function, as we can infer from Eq. (4.18) and therefore is not important in this framework.

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